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**RAK-003-001408** Seat No. ________

# B. Sc. (Sem. IV) (CBCS) Examination

**March / April - 2019**

**BSMT-401(A) : Title of Course :**

**Advance Calculus & Linear Algebra**

**Faculty Code : 003**
**Subject Code : 001408**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

**Instructions:**

a. All questions are compulsory.
b. Right hand side digit indicates the mark.

1. Answer the following questions in short: [20]

(1) If $u(x,y,z) = x^2yz + xy^2z + xyz^2$, then find $\dfrac{\partial u}{\partial z}$.

(2) $w = \frac{y}{x} + \frac{x}{z} + \frac{z}{x}$, then $x\dfrac{\partial w}{\partial x} + y\dfrac{\partial w}{\partial y} + z\dfrac{\partial w}{\partial z} =$ ________.

(3) If $v = \tan^{-1}\left(\frac{xy}{x^2+y^2}\right)$, then $x^2\dfrac{\partial^2 v}{\partial x^2} + 2xy\dfrac{\partial^2 v}{\partial x \partial y} + y^2\dfrac{\partial^2 v}{\partial y^2} =$ ________.

(4) Total differential d$z$ at $(x,y)$ of $z = x^2 + y^2$ is ________.

(5) For $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

(6) $\frac{\partial(u,v)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(u,v)} =$ ________.

(7) Find $\nabla\phi$ if $\phi = x^3 + y^3 + z^3$.

(8) For a vector point function $\overline{f}, div(curl\ f) =$ ________.

(9) Write Laplace equation for scalar point function $\phi$.

(10) $\int_0^1\int_0^1 (x+y)\mathrm{d}x\mathrm{d}y =$ ________.

(11) $\int_0^a\int_0^b\int_0^c xyz\ \mathrm{d}z\mathrm{d}y\mathrm{d}x$ ________.

(12) Find $\int_C \frac{\mathrm{d}x}{x+y}$, where $C : x = at^2, y = 2at, 0 \le t \le 2$.

(13) $\int_C P dx + Q dy$ is independent of path if, ________.

(14) In the surface integral $\iint_S \overline{V} \cdot \overline{n} d\sigma$, $\overline{n} =$ ________.

(15) For integer $r$, $\Gamma(r+1) =$ ________.

(16) $\beta(2,2) =$ ________.

(17) $V$ is an inner product space and $\overline{u}, \overline{v} \in V, \alpha \in \mathbb{R}$, then $||\alpha\overline{u}|| =$ ________

(18) If $\overline{u}$ and $\overline{v}$ are orthogonal elements of inner product space $V$, then $< \overline{u}, \overline{v} >=$ ______ .

(19) For any inner product space, $||\overline{u} + \overline{v}||^2 + ||\overline{u} - \overline{v}||^2 =$ ________.

(20) Define: Homogeneous Function.

2. (a) Attempt any **three** [06]

(1). If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}; & (x,y) \neq (0,0) \\ = 0; & (x,y) = (0,0) \end{cases}$, then find $f_x(0,0)$ and $f_y(0,0)$.

(2). If $f(x,y) = x^2y - 3y$, then find the approximate value of $f(5.12, 6.85)$.

(3). If a vector function $\overline{F} = (x^2z - axyz)\hat{i} + (xy - 3x^2yx)\hat{j} + (yz^2 - xz)\hat{k}$ is solenoidal, then find the value of $a$.

(4). If $c$ is any real number and $\phi$ & $\psi$ are differential scalar functions on the domain $D$ of $\mathbb{R}^3$, then show that

(i) $\nabla(c) = 0$

(ii) $\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$.

(5). If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$.

(6). If $f(x,y) = \sin^{-1}\left(\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}\right)$, then show that $\frac{f_x}{f_y} = -\frac{y}{x}$.

(b) Attempt any **three** [09]

(1). Expand $e^x \cos y$ in powers of $x$ and $y$ up to three degree.

(2). If $\overline{f} = (x^3, y^3, z^3)$, then prove that $curl\ \overline{f} = \overline{0}$ and $grad\ (div\ \overline{f}) = 6\overline{r}$.

(3). Prove that $curl(\phi\overline{f}) = \phi\ curl(\overline{f}) + grad\ (\phi) \times \overline{f}$.

(4). If $y^3 - 3ax^2 + x^3 = 0$, then prove that $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0$.

(5). If $f(x,y) = \sin^{-1}\sqrt{\frac{x^3+y^3}{x^2+y^2}}$, then show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{\tan z}{2}$

(6). Show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = 8xyz$, if $u = e^{x^2+y^2+z^2}$.

(c) Attempt any **two** [10]

(1). Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipse
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$

(2). State and prove Euler's theorem.

(3). If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right), x\neq y$, then show that

$(i)\ x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

$(ii)\ x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(1-4\sin^2 u)\sin(2u).$

(4). State and prove Taylor's theorem for two variable.

(5). If $\overline{F}=grad(x^3+y^3+z^3-3xyz)$, then prove that $grad(div\ \overline{F})=6(\hat{i}+\hat{j}+\hat{k})$.

3. (a) Attempt any **three** [06]

(1). Change the order of integration in $\int_0^a\int_y^a f(x,y)dxdy$.

(2). Find the value $\int_0^1\int_0^{x^2}\int_0^{x+y}(2x-y-z)dzdydx$

(3). Find $\int_{(1,1)}^{(2,3)} xdy$.

(4). Obtain the area of ellipse using Green's theorem.

(5). For $p>0, q>0$, prove that $\beta(p,q)=\beta(q,p)$.

(6). Define : *Orthogonal basis, Orthonormal basis.*

(b) Attempt any **three** [09]

(1). Change the order of integration in the integral $\int_0^1\int_{\frac{x^2}{a}}^{2a-x} xydydx$ and hence evaluate it.

(2). If parametric equations of $C$ are $x=1+t, y=t^2$, where $0\leq t\leq 1$, then find $\int_C (x^2-y^2)dx$ and $\int_C (x^2-y^2)dy$.

(3). In Euclidean inner product space $\mathbb{R}^3$, transformation the basis
$S=\{(1,0,0),(3,7,-2),(0,4,1)\}$

into an orthonormal basis using Gram Schmidt process.

(4). Evaluate $\iint[xy(1-x-y)]^{\frac{1}{2}}dxdy$, taken over the area of the triangle with sides $x=0, y=0, x+y=1$.

(5). Prove that $\int_0^1 \frac{x^5}{\sqrt{1-x^4}}dx = \frac{\pi}{8}$.

(6). Prove that the line integral $\int_{C(0,0)}^{(x,y)} (6xy^2 - y^3)dx + (6x^2y - 3xy^2)dy$ is independent of path and also obtain its value.

(c) Attempt any **two** [10]

(1). If $\overline{V} = y\hat{i} + z\hat{j}$ and $S$ is a part of $2x + 2y + z = 2$, then find $\iint_S V_n d\sigma$.

(2). Change in cylindrical co-ordinate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{1+x+y} (x^2 - y^2)dxdydz$ and hence evaluate it.

(3). For $u = (u_1, u_2, u_3), v = (v_1, v_2, v_3) \in \mathbb{R}^3$, prove that $u \cdot v = 2u_1v_1 + u_2v_2 + 4v_3v_3$ is inner product on $\mathbb{R}^3$.

(4). For $p > 0, q > 0$, prove that $\beta(p, q) = \frac{\Gamma p \, \Gamma q}{\Gamma p+q}$.

(5). Verify the divergent theorem for $\overline{V} = x\hat{i} + y\hat{j} + z\hat{k}$ and $S$ is a sphere $x^2 + y^2 + z^2 = 1$.

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